FREE-CONVECTIVE HEAT TRANSFER AT SMALL VALUES OF THE PARAMETER GrPr

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Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 6, pp. 703–709, 1966

UDC 536.24

Heat transfer to bodies of different shapes in an infinite volume is examined. The existing experimental results are analyzed. In addition, results of an experimental investigation of heat transfer using the calorimetric method and the electrothermal analogy are also presented.

When heat transfer depends appreciably on the geometry of the heat-emitting body, the conditions correspond to values of the parameter $GrPr < 10^4$. The heat transfer law $Nu = f(GrPr)$ then deviates from the 1/4 power law. This is illustrated by the graph of Fig. 1.

The conditions examined may occur in practice either in a rarefied gas or for bodies with small characteristic dimensions (microorganisms and dust), or again in heat transfer at small temperature differences or small body forces (under weightless conditions). The investigations which have been conducted [1–9] correspond in the main to this range of conditions. Some of the relations are presented in Fig. 2. With regard to the results of Nesenchuk for a cube, sphere, cylinders, and a parallelipiped, it should be noted that they give an increased value of the heat transfer coefficient in comparison with the repeatedly confirmed results for a sphere and for cylinders.

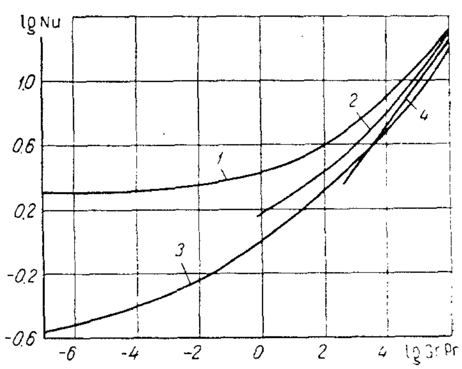

Fig. 1. Experimental relations for heat transfer in gases over a wide range of pressures: 1) for a sphere, according to Elenbaas [1]; 2) for vertical plates, according to Saunders [2]; 3) for horizontal cylinders, author's generalization [5]; 4) the Lorentz relation $Nu = 0.54(GrPr)^{1/4}$.

Analysis of the experimental results (Figs. 1 and 2) permits us to draw the following conclusions:

1. A lessening of the role of convective heat transfer leads to increase of the influence of body shape on heat transfer. For example, the heat emission from spheres may be several times greater than that from cylinders or plates with identical characteristic dimensions.

2. The strong influence of body shape forbids the description of heat transfer in the region of small values of GrPr by a single relation, even with a special choice of characteristic dimension.

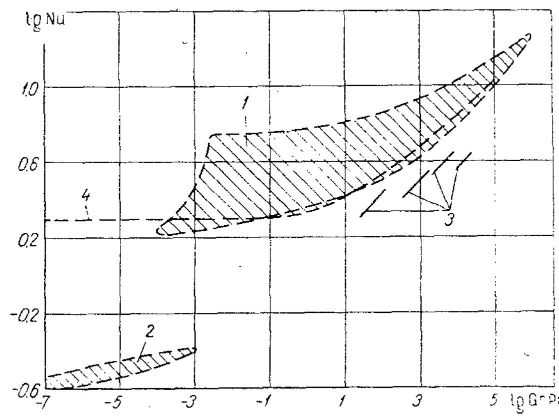

Fig. 2. Correlation of test data: 1) Nesenchuk [4]; 2) Hauser [7]; 3) Semyashkin [8]; 4) Kyte, Madden, and Piret [9].

3. The absence of published results of systematic experiments with bodies of different shapes makes it impossible to give definite recommendations for calculating heat transfer to bodies of complex shape.

4. In the subsequent analysis it is convenient to use the known fact (see, for example, [10]) that as ($GrPr \to 0$) for spheres $Nu \to 2$, and for circular cylinders $Nu \to 0$. The values $Nu \approx 2$ for spheres occurs even at $GrPr \approx 0.1$.

An attempt is made in the present paper to fill in the existing gap in the experimental investigations. The results given below are for a calorimetric study of heat transfer with finite vertical cylinders in vacuum, as well as the results of an investigation of heat transfer by the electrothermal analogy method.

Calorimetric investigations. The experimental equipment described in [5] was used to investigate heat transfer on a series of finite cylinders with ratio $l/d = 4$ and diameters of 12, 16, 20, and 28 mm in the pressure range $1 \cdot 10^{-2}$–760 mm Hg. The temperature of the specimens did not exceed 100° C, while the shell of the vacuum chamber was kept at room temperature. Thermal radiation effects were excluded on the basis of the results of experiments in vacuum (at a pressure of the order $2 \cdot 10^{-5}$ mm Hg), when molecular heat transfer may be neglected.

Heat transfer on the cylinders investigated as described by a single relation (Fig. 3) in the range $GrPr = 10^{-4}$–10^4, while $Nu \approx const = 1.06$, for $GrPr < 10^{-5}$

which is evidence of the absence of a free convection effect on heat transfer. The cylinder diameter was taken as the characteristic dimension. The relation $Nu = f(GrPr)$, derived from Fig. 3 is characteristic of heat transfer with bodies of finite dimensions and for cylinders of infinite length in a shell [5].

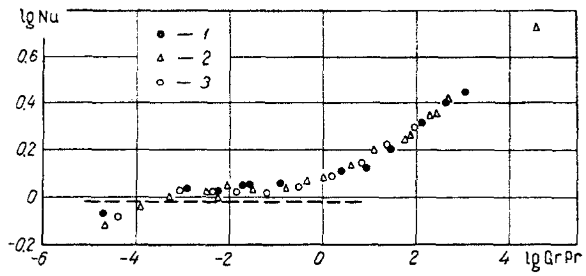

Fig. 3. Heat transfer on finite vertical cylinders with $l/d = 4$ and temperature difference $\Delta t = 80°$ C 1) for d = 20 mm; (2) 16; 3) 12; -- heat transfer on a cylinder with $l/d = 4$ in the absence of free convection (from electro-thermal analogy data).

The constancy of the value of Nu is due to two essentially different causes: i.e., (a) the influence of the shell walls, leading to a lessening of free convection, and (b) the approximation of the heat transfer conditions to those of pure thermal conduction in an infinite volume.

The combination of effects of different kinds makes analysis difficult. On the other hand, a detailed quantitative investigation of heat transfer at small values of GrPr for bodies of complex shape by the calorimetric method requires the carrying out of a large number of experiments, the labor of which is made worse by the measurement of very small heat fluxes.

A quantitative analysis is appreciably facilitated by the results of an investigation of heat transfer by the electrothermal analogy method.

Electrothermal analogy. The object of the experimental investigations using the electrothermal analogy is to obtain data on heat transfer in the region of very small values of GrPr. The limiting case of heat transfer with GrPr → 0 is pure heat conduction in an infinite volume. Then the temperature field, like any potential field, is described by the Laplace equation. The transfer conditions are formulated mathematically as follows.

In the space $\Delta\varphi = 0$ at an infinite distance from the body surface

$$\varphi|_\infty = 0, \quad \frac{\partial\varphi}{\partial x}\bigg|_\infty = \frac{\partial\varphi}{\partial y}\bigg|_\infty = \frac{\partial\varphi}{\partial z}\bigg|_\infty = 0,$$

on the body surface

$$\varphi|_k = \varphi_0 = f(x,\ y,\ z), \qquad (1)$$

$$\frac{\partial\varphi}{\partial n}\bigg|_k = \psi(x,\ y,\ z). \qquad (2)$$

In dimensionless form the boundary conditions at an infinite distance are

$$\bar{\varphi}|_\infty = 0, \quad \frac{\partial\bar{\varphi}}{\partial x}\bigg|_\infty = \frac{\partial\bar{\varphi}}{\partial y}\bigg|_\infty = \frac{\partial\bar{\varphi}}{\partial z}\bigg|_\infty = 0,$$

and on the surface

$$\frac{\partial\bar{\varphi}}{\partial\bar{n}}\bigg|_k = \frac{l}{\varphi_0}\,\psi(x,\ y,\ z). \qquad (3)$$

It follows from (1), (2), and (3) that the dimensionless gradient along the normal is a function only of the geometry of the system. On the other hand,

$$\frac{\partial\bar{\varphi}}{\partial\bar{n}} = \frac{il}{\varphi_0\sigma},$$

where i is the local specific flux, l is a characteristic dimension of the system, and σ is a transfer coefficient. Therefore, $\frac{\partial\bar{\varphi}}{\partial\bar{n}}$ has the significance of a local dimensionless transfer coefficient and is the Nusselt number Nu_T for heat conduction and for electrical conduction its electrical analog Nu_E. For identical system geometry

$$Nu_T = Nu_E.$$

The mean value

$$Nu = \frac{1}{F}\int_F \frac{\partial\bar{\varphi}}{\partial\bar{n}}\,dF$$

is easily determined experimentally, since it may be expressed in terms of the parameters of the electric circuit,

$$Nu = Il/F\,\Delta V\,\varkappa = l/FR\,\varkappa.$$

The essence of the experimental method of determining Nu lies in determining the resistance R for a known electrical conductance $\varkappa$.

The specimens used in the experiments – spheres, finite cylinders, and finite plates – were located at the center of a metal vessel of dimensions 250 × 250 × 300 mm filled with an electrolyte, an unsaturated solution of boric acid in distilled water.

A container of this size was chosen so that for the selected specimens (Table 1) the shell of the container could have no appreciable influence on the form of the potential field at the specimen surface. In that case the shape of the jacket has no significance.

The liquid chosen as electrolyte should have a conductivity such that difficulties in measuring the electrical resistance by means of the usual laboratory instruments may be avoided. This requirement was fully satisfied by an aqueous solution of boric acid with conductivity $1.047 \cdot 10^{-4}$ ohm^{-1} cm^{-1}. The latter was measured before each test.

The specimen and the shell of the container served as electrodes. The resistance between them was measured in the experiment by means of a bridge supplied with ac power from an af generator [11] to avoid polarization. The null detector used was a cathode-ray oscillograph type SI-1. The specimens were chosen with one fixed characteristic dimension (3 mm), used as a control dimension in the calculations.

Table 1 shows data on the specimens and the results of measurements and of calculations of Nu number, while Fig. 4 gives the variation of Nu number for cylinders and plates in the range $l/d = 1–20$. The result for the sphere is a standard. For the geometric dimensions of the shell used in the test, the theoretical value of Nu lies in the range 2.031–2.062. These quantitative data correspond to transfer of heat or electricity from the sphere being tested to a concentric shell with a diameter whose value falls between the diameters of spheres circumscribed about and inscribed within the container shell. In the experiment the value Nu = 2.03 ± 1.5% was obtained. Hence it follows that the conditions of heat transfer were simulated with an error not exceeding 1.5%.

Thus, bodies of infinite dimensions of any convex shape have a nonzero numerical limit of Nu value for free convection. This value is determined not by the film regime of heat transfer, as has previously been stated, but by heat transfer in the pure heat conduction regime. If the smaller linear dimension of a convex body is taken as the characteristic value, the value of the parameter Nu for different bodies lies in the range between the relations for heat transfer on a sphere and a circular cylinder of infinite length.

On the basis of the experimental data obtained (Fig. 4), the relations $Nu = f(l/d)$ for cylinders and plates of the shapes examined may be represented in the form

$$Nu = 1.58\,(l/d)^{-0.32} \qquad (4)$$

for cylinders, and

$$Nu = 2\,(l/d)^{-0.32} \qquad (5)$$

for plates, with an error not exceeding ± 5%. These formulas can be used for engineering calculations when $l/d < 20$.

From Fig. 3 we may make a comparison of the results of measurement of heat transfer by the calorimetric method and the electrothermal analogy method for finite cylinders with $l/d = 4$. The fact that the values increased by 10% on the average for the experiments in vacuum may be explained by the influence of the shell.

In approximate calculations formulas (4), and (5) can be used with sufficient accuracy even at values of GrPr when the Nu number found from formulas (4) and (5) is larger than the Nu value determined from the relation for cylinders of infinite length.

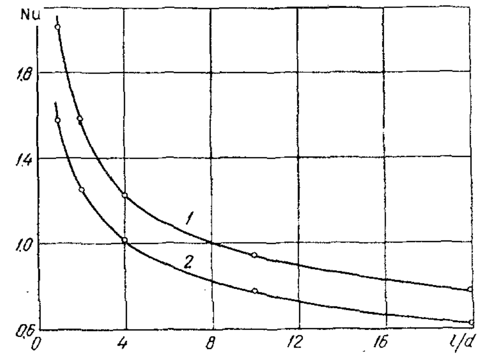

Fig. 4. Heat transfer in the absence of free convection; 1) plate (d – – width, l ––length; it is assumed that the plate has no thickness, i.e., that the plate thickness is not its characteristic dimension); 2) cylinders (d –– cylinder diameter, l –– length).

For cylinders and plates with $l/d \gg 1$, and also for other shapes, use may be made of the results of computation of steady electrical fields [12, 13]. Limiting values of Nu may be obtained on the basis of the analogy between the temperature field and the electric field in a conductor and the electrostatic field in a dielectric. The value of Nu_E for an electric field was found earlier. In the same way the electrostatic analog of the Nu number may easily be determined,

$$Nu_{es} = Cd/\varepsilon F.$$

Table 2 shows limiting values of Nu for bodies of different shapes. Some of these are already well known in the heat transfer literature [10]. The values for the capacitance of ellipsoids are taken from [12]. Their surface areas in the expression for Nu were found by integration.

For other shapes (prisms, oval cylinders, etc.) the results of the present paper may be used as an estimate.

In analyzing heat transfer for bodies of complex shape under conditions corresponding to small values of the parameter GrPr, it is reasonable to introduce some function of the dimensionless body shape. In the

Table 1

Values of Nu for Bodies of Various Shapes

Specimen	Characteristics of specimen: d or h, mm	l, mm	F, mm²	R_x±50, ohm	Nu*	lg Nu	l/d
Sphere	3		28.3	5000	2.03 ±0.03	0.307	
Cylinder	3	3	42.4	4600	1.57 ±0.027	0.196	1
		6	71	3460	1.25 ±0.021	0.097	2
		12	127.2	2370	1.012±0.015	0.005	4
		30	297	1340	0.768±0.011	−0.116	10
		60	580	850±30	0.62 ±0.008	−0.207	20
Plate of thickness 0.1 mm	3	3	19.2	6950	2.13 ±0.09	0.328	1
		6	37.8	4800	1.58 ±0.049	0.198	2
		12	75	3140	1.22 ±0.026	0.0865	4
		30	186.6	1650	0.935±0.016	−0.03	10
		60	372.6	1000±30	0.77 ±0.011	−0.114	20

Table 2

Limiting Values of Nu

Shape	Formula for C	Reference	Formula or value for Nu
Sphere in infinite volume			2
Coaxial cylinders with diameters d and D of infinite length			$\dfrac{2}{\ln \dfrac{D}{d}}$
Circular plate		[10]	$\sim 2,55$
Single cylinder of length l; $l \gg d$		[10]	$\sim \dfrac{2}{\ln \dfrac{l}{d}}$
Strip of width d and length l; $l \gg d$		[10]	$\sim \dfrac{\pi}{\ln \dfrac{4l}{d}}$
Cube of side d	$C \sim 0.65 \cdot 4\pi\varepsilon\, a$	[13]	$\sim 1,36$
Elongated ellipsoid of revolution with diameter of equatorial section d and eccentricity of meridional section e;	$C = \dfrac{4\pi\varepsilon\, de}{\ln \dfrac{1+e}{1-e} \sqrt{1-e^2}}$	[12]	$\dfrac{8e}{\ln \dfrac{1+e}{1-e} \left(\sqrt{1-e^2} + \dfrac{\arcsin e}{e} \right)}$
Oblate ellipsoid of revolution with diameter of equatorial section d and eccentricity e.	$C = \dfrac{2\pi\varepsilon\, de}{\arcsin e}$	[12]	$\dfrac{4e}{\arcsin e \left(1 + \dfrac{1-e^2}{e} \ln \dfrac{1+e}{\sqrt{1-e^2}} \right)}$

simplest case this will be the ratio of the characteristic dimensions.

In choosing a "characteristic" dimension we may take into consideration the following argument. If the characteristic dimensions of the body are markedly different, for example, if the dimensions differ by more than two orders of magnitude, then the influence of one of them on the heat transfer will be enhanced. Thus, for cylinders, when the length is increased without bound, its influence declines and becomes negligibly small. Similarly, for a thin plate of finite dimensions, an unbounded decrease of its thickeness has no appreciable effect on variation of heat transfer. In estimating the mean values of the heat transfer coefficient, or in analyzing experimental data on heat transfer on bodies of complex shape, it should first be established which of the simplest shapes (spheres, cylinder or plate) corresponds best to the body under examination. As a characteristic dimension we may then take, respectively, the diameter of a sphere or a cylinder or the width of a plate. With this choice of "characteristic" dimension it is easy to compare heat transfer on the body being tested with the widely known results on heat transfer for spheres, infinite cylinders, and also with the results of the present paper.

We shall not concern ourselves here with the question of the influence of temperature jump and slip at large Knudsen numbers. An approximate method of computing heat transfer in such conditions for bodies of very simple shape was proposed in [5].

NOTATION

Nu, Gr, Pr are the Nusselt, Grashof, and Prandtl numbers: φ is the potential function; n is the coordinate along normal to surface: I is the current; l is the characteristic dimension of body: F is the specimen surface area: C is the capacity of body in an infinite volume: ε is the dielectric constant of medium surrounding body; d is the characteristic dimension.

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10 June 1966

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